Tagore Public School, Shastri Nagar, Jaipur

Worksheet of Mathematics (041)
Holiday Homework for Class XII
Session-(2017-2018)

- ***Solve the problems in a separate copy and submit the homework compulsorily by 03/01/2018***

Questions are as follows:-

1. Find the matrix A satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.
2. Show that A = $\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ $satisfies\ the\ equation\ A^2 - 3A - 7I = 0\ and\ hence\ find\ A^{-1}$.
3. If A = $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}\ and\ C = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$ verify (i) (AB)C=A(BC) (ii) A(B+C) = AB+AC.
4. Using properties of determinants prove that $\begin{vmatrix} y^2z^2 & yz & y+z \\ z^2x^2 & zx & z+x \\ x^2y^2 & xy & x+y \end{vmatrix}$=0.
5. If A+B+C=0, then prove that $\begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix}$ =0.
6. If the coordinates of the vertices of an equilateral triangle with sides of lengths a are $(x_1, y_1), (x_2, y_2), (x_3, y_3)\ then\ prove\ \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = \frac{3a^4}{4}$.
7. Check the continuity at the indicated points:-
$$f(x) = \begin{cases} \dfrac{e^{1/x}}{1 + e^{1/x}}, & if\ x \neq 0 \\ 0, & if\ x = 0 \end{cases} \quad at\ x = 0.$$
8. Find the value of k, if f is continuous at x = 2, $f(x) = \begin{cases} \frac{2^{x+2} - 16}{4^x - 16}, & x \neq 2 \\ k, & if\ x = 2 \end{cases}$.
9. Examine the differentiability of f where f is defined by $f(x) = \begin{cases} x[x], & if\ 0 \leq x < 2 \\ (x-1)x, & if\ 2 \leq x < 3 \end{cases}$ at x=2.
10. Using M.V.T prove that there is a point on the curve $y = 2x^2 - 5x + 3$ between the points A(1,0) and B(2, 1) where tangent is parallel to the chord AB. Also find the point.

11. A kite is moving horizontally at a height of 151.5 metres. If the speed of the kite is 10 metres/sec, how fast is the string being let out when the kite is 250 metres away from the boy who is flying the kite? The height of the boy is 1.5 metres.
12. Two men A and B start with the velocities V at the same time from the junction of two roads inclined at 45^0 to each other. If they travel by different roads find the rate at which they are being separated.
13. Find the maximum value of $\left(\frac{1}{x}\right)^x$.
14. Find the stationary point for $f(x) = x^x$.
15. Find the least value of the function $f(x) = ax + \frac{b}{x}$ (where a>0, b>0, x>0) .
16. A manufacturer of electronic circuits has a stock of 200 resistors, 120 transistors and 150 capacitors and is required to produce two types of circuits A and B. Type A requires 20 resistors, 10 transistors and 10 capacitors. Type B requires10 resistors, 20 transistors and 30 capacitors. If the profit on type A circuit is Rs 50 and that on type B circuit is Rs 60, formulate this problem as a LPP so that the manufacturer can maximise his profit.
17. Find the area of the region bounded by the curve y=$x^3, y = x + 6$ and $x = 0$.
18. Find the area of the region bounded by the parabola $y^2 = 2px$ and $x^2 = 2py$.
19. If $\vec{a}, \vec{b}$ and $\vec{c}$, determine the vertices of a triangle. Show that $\frac{1}{2}[\vec{b} X \vec{c} + \vec{c} X \vec{a} + \vec{a} X \vec{b}]$ give the vector area of the triangle. Hence deduce the condition that the three points $\vec{a}, \vec{b}$ and $\vec{c}$ are collinear. Also find the unit vector normal to the plane of the triangle.
20. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\jmath} - \hat{k}$, find a vector c such that $\vec{a} X \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$.
21. Two systems of rectangular axis have the same origin. If a plane cuts them at distances a, b, c and a', b', c' respectively from the origin, prove that $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}$.
22. If a variable line in two adjacent positions has direction cosines l, m, n and $l + \delta l, m + \delta m, n + \delta n$, show that the small angle $\delta\theta$ between the two positions is given by $\delta\theta^2 = \delta l^2 + \delta m^2 + \delta n^2$.
23. Maximize and minimize Z = 3x – 4y subject to x - 2y ≤ 0, -3x + y ≤ 4, x – y ≤ 6 and x,y ≥ 0.
24. A letter is known to have come either from TATANAGAR or from CALCUTTA. On the envelope, just two consecutive letters TA are visible. What is the probability that the letter came from TATANAGAR?
25. A bag contains (2n+1) coins. It is known that n of these coins have a head on both sides whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, determine the value of n.
26. Suppose, a random variable X follows the Binomial distribution with parameters n and p, where 0< $p < 1$. If $\frac{P(x=r)}{P(x=n-r)}$ is independent of n and r then find p.
27. For a loaded die, the probability of outcomes are given as under: P(1)=P(2)=0.2, P(3)=P(5)=P(6)=0.1 and P(4)=0.3. The die is thrown 2 times. Let A and B be the events, “Same number each time”;”A total score is 10 or more” respectively. Determine whether or not A and B are independent.

28. A bag contains 5 red marbles and 3 black marbles. Three marbles are drawn one by one without replacement. What is the probability that at least one of the three marbles drawn be black, if the first marble is red.
29. In a dice game a player pays a stake of Rs. 1 for each throw of a die. She receives Rs. 5 if the die shows a three, Rs. 2 if the die shows 1 or 6 and nothing otherwise. What is the players expected profit per throw over a long series of throws?
30. Evaluate:-

   i. $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}}\, dx.$

   ii. $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}\, dx.$

   iii. $\int \frac{dx}{x\sqrt{x^4-1}}.$

   iv. $\int \frac{x^{1/2}}{1+x^{3/4}} dx$

   v. $\int_0^{\frac{\pi}{2}} \frac{\tan x}{1+m^2 tan^2 x} dx$

   vi. $\int_0^{1/2} \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

   vii. $\int e^{-3x} cos^3 x dx$

   viii. $\int \sqrt{tanx}\,$dx

   ix. $\int_{-\pi/4}^{\pi/4} log|\sin x + \cos x|\, dx$

   x. $\int_0^{\frac{\pi}{2}} \sqrt{1-\sin 2x}$dx.

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